A TEST CASE FOR PROBABILISTIC RELIABILITY ASSESSMENT USING SIMULATION TECHNIQUES AND FEM

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Abstract: A numerical example arising from probabilistic reliability assessment of a large-scale finite element model of a structure using the Monte Carlo method and Importance Sampling will show the efficiency of iterative solvers for problems with multiple right-hand sides. The CALFEM toolbox is used as a tool for the finite element analysis.

Keywords: simulation techniques, variance reduction techniques, uncertainty analysis, iterative solvers

1 INTRODUCTION TO SIMULATION BASED PROBABILISTIC RELIABILITY ANALYSIS

It is a well known fact that the structural reliability assessment process is actually still based on conservative approaches, such as single safety factor or partial reliability factors approaches, which are from designers' point of view deterministic (Marek, 2003). However, the rapid development of computer technology allows designers to use simulation techniques for full probabilistic reliability assessment in structure engineering for some cases; see for instance the Simulation Based Reliability Assessment method (SBRA) (Marek, 2003).
This contribution sums up our experiences with development and testing of the own software tool for **automated** structural reliability assessment of general systems.
The main aim is to examine the applicability of several mathematically equivalent simulation techniques, namely Direct Monte Carlo, and Importance Sampling to the process of the probabilistic design.
The presented numerical experiments indicate a pronounced variance reduction when Importance Sampling is applied. In all cases, the simulation process ran automatically in the SBRA environment, that is, without claiming additional a-priori information about the structural behavior of a simulated problem.

2 THE UNCERTAINTY MODELING USING THE SBRA METHOD

In SBRA method, the randomness of input parameters is described by general discrete random variables. The character of distributions is derived by expert opinions (random loads) and/or laboratory experiments (mechanical characteristics of materials).

The reliability assessment is powered by the Monte Carlo simulation technique with the variance reduction technique based on Importance Sampling.
The Importance Sampling method uses a new density function f^* for sampling which is called an importance sampling density function. The purpose of this approach is to increase the frequency of occurrence of rare events (failures), even when the total number of simulation steps remains low (Fishman).

3 PROBABILISTIC RELIABILITY ASSESSMENT

Let the randomness of input parameters $X=(x_1,\ldots,x_m)$ is described by the density function $f(X)$. Let $G(X)$ is the limit state function and $G(X)<0$ indicates a failure of the system. The aim is to evaluate the reliability integral

$$P_f = \int_{G(X)<0} f(X)dX \ .$$

For realistic problems, the reliability integral is difficult to compute because (i) the dimension of the parameter space m is large, (ii) the probability of failure P_f is very close to zero, and (iii) the evaluation of the limit state function $G(X)$ involves often a time-expensive analysis.

4 AUTOMATED VARIANCE REDUCTION TECHNIQUES BASED ON IMPORTANCE SAMPLING

The uniform density function is used as the importance sampling density function (Fishman 1997). Although this approach does not require any a-priori information about the structural behavior of the analyzed system, a significant variance reduction has been observed (Praks, Brožovský, 2005).

4 THE PROBABILISTIC RELIABILITY ANALYSIS TEST CASE

Consider a concrete frame subjected to a six uniformly distributed loads (Fig. 1), for details see (Praks, Brožovský, 2005). All loads are assumed to be mutually independent normal random variables, see Tab. 1. In our model, the safety function has the form

$$Z = R - S,$$

where the symbol R denoted concrete tensile strength described by normal random variable with parameters R = 1 ± 0.1 MPa. The symbol S denoted the maximum value of the main principal stress of an element of the structure.
The computation of probability of failure $P_f=P(Z<0)$ was powered by the simulation approach. The frame is discretized using the finite element method by the CALFEM toolbox. Because of the fact that we assume stochastic character of loads in our model, the stochastic contribution of random loads will influence only the right hand side vectors of the linear system of equations. In the numerical example (Praks, Brožovský 2005), we computed 1 000 simulation steps, so the corresponding multiple system of linear equations had 1 000 right hand sides and the total number of unknowns was 16 188 × 1 000 = 16 188 000. As the solver of this multiple linear system of equations we used the SBCG algorithm (Feng 1995, Praks 2005). Let us notice that the solution of one linear system by the classical PCG required 205 matrix-vector operations. To solve all 1 000 linear systems of equations, only 446 matrix-vector operations were needed. The total solution time was 9.1 minutes on an NB Premio 5050N with 1 GB RAM.
The aim of this example was to find a distribution of main principal stresses of elements in the structure. For computation of element stresses Es = (σ_x; σy; τxy) from the element displacement vector we used the Calfem call `'planrs'`. Then we calculated for each element in domain the maximum and the minimum values (variables denoted here as σ_1; σ_2) of the main principal stress in the following way:

$$sp = 0.5(\sigma_x + \sigma_y + sqrt((\sigma_x - \sigma_y)^2 + 4(\tau xy)^2)$$
$$sm = 0.5(\sigma_x + \sigma_y - sqrt((\sigma_x - \sigma_y)^2 + 4(\tau xy)^2)$$
$$\sigma_1 = max(sp; sm)$$
$$\sigma_2 = min(sp; sm)$$

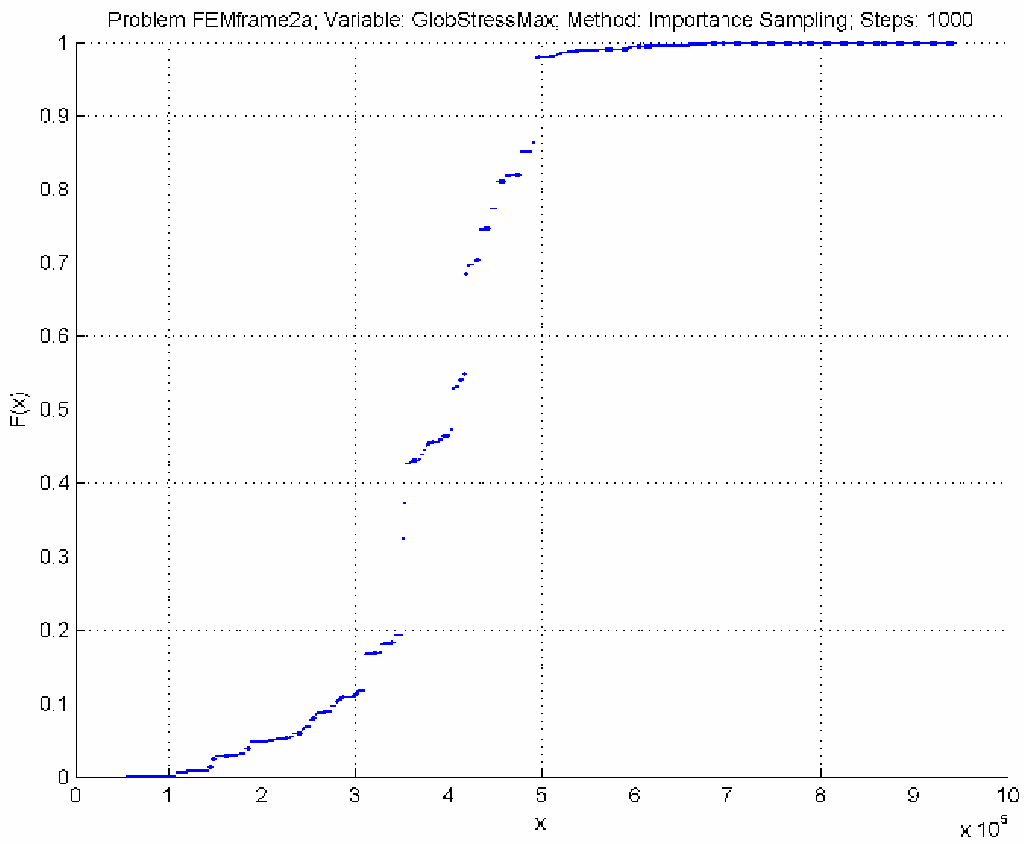

Fig. 2. The distribution of the maximal principal stress σ_1 in the frame test case estimated by the direct Monte Carlo.

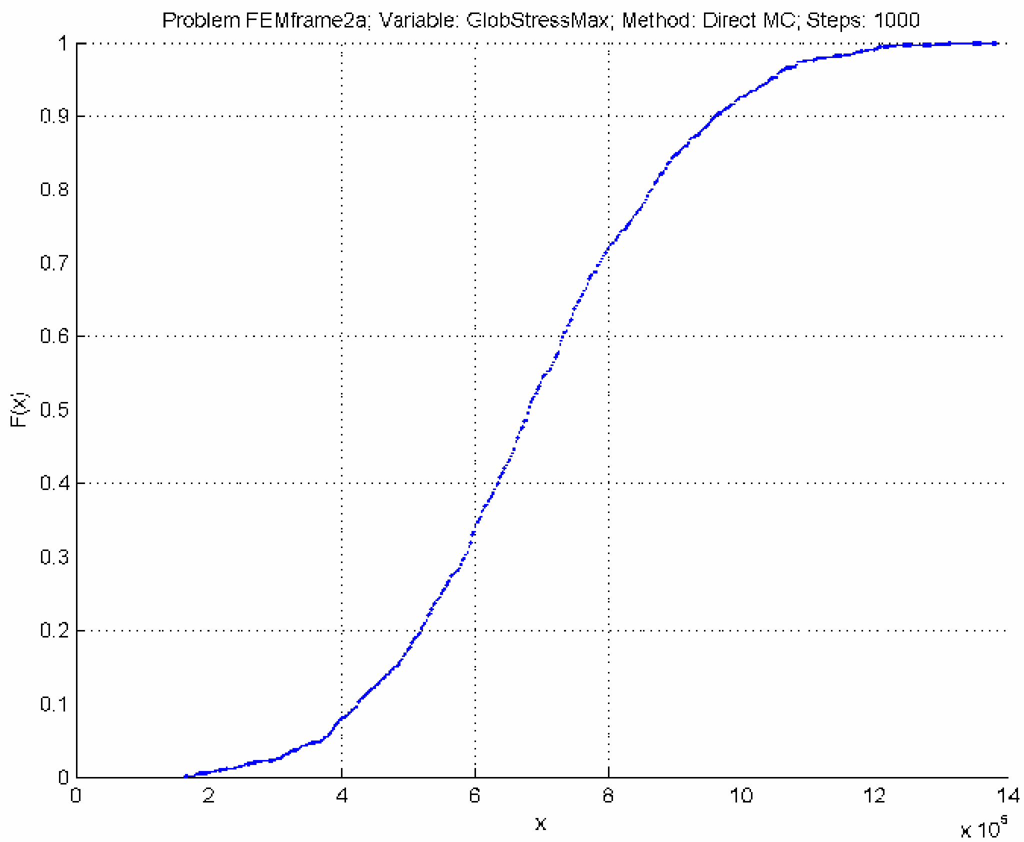

Fig. 3. The distribution of the maximal principal stress σ_1 in the frame test case estimated by the Importance Sampling.

When Importance Sampling was applied, the probability of failure was estimated as P_f = 0.0390. Let us accent that the Importance Sampling approach benefit the detection of low probability (critical) events. Failures will tend to occur when random parameters hold extreme values, usually in “tail” regions. The presented algorithm based on Importance Sampling is able to detect low probability events when the total number of simulation steps remains low, see Fig. 2 and Fig. 3. In future work we would like to solve real 3D large scale applications and dynamic reliability problems (Briš, Praks, 2005).

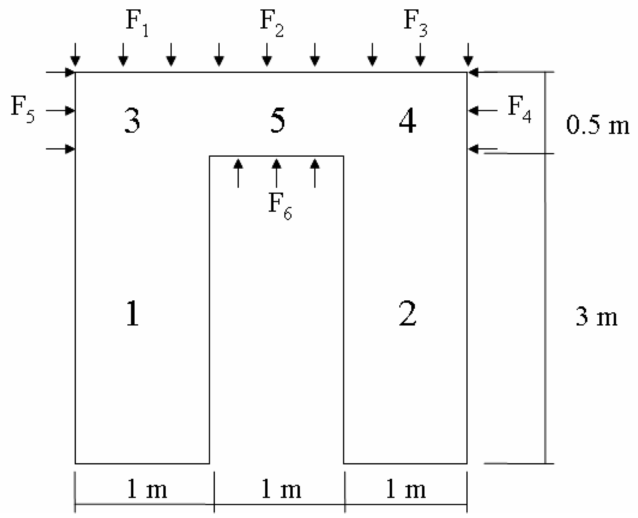

Fig. 1. The model problem for the probabilistic reliability assessment. The geometry of the frame contains 5 subdomains denoted by symbols 1, 2, . . . , 5.

Table 1 Parameters of the random loads.

Forces	Mean	Std. Dev
F_1	15 kN	5 kN
F_2	15 kN	5 kN
F_3	15 kN	5 kN
F_4	4 kN	4 kN
F_5	4 kN	4 kN
F_6	0 kN	4 kN

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